Transformation of Equation

Roots with sign negative:→ To transform an equation into another equation whose roots are the roots of the given equation with sign changed (with contary sign).

Sol^n:→

Let $\alpha_1, \alpha_2, \dots \alpha_n$ be roots of the equation.

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \dots + P_{n-1} x + P_n = 0$$

$$\therefore x^n + P_1 x^{n-1} + P_2 x^{n-2} + \dots + P_{n-1} x + P_n \equiv (x-\alpha_1)(x-\alpha_2) \dots (x-\alpha_n)$$

changing x by $(-y)$. We have whenever n is even or odd.

$$y^n - P_1 y^{n-1} + P_2 y^{n-2} - \dots \pm P_{n-1} y + P_n \equiv$$

$$\cancel{(y+\alpha_1)(y+\alpha_2)} \dots \cancel{(y+\alpha_n)}$$

$$(y+\alpha_1)(y+\alpha_2) \dots (y+\alpha_n)$$

so the polynomial

$$y^n - P_1 y^{n-1} + P_2 y^{n-2} - \dots + (-1)^n P_n = 0$$

Q. Therefor an equation whose roots are $-\alpha_1, -\alpha_2, \cdots\cdots, -\alpha_n$ and to effect the required transformation we have only to

"changed sign of every alternative term of the given equation beginning with the second".

If the given equation is compleate. (If not it may be made to take that from by supplying missing terms with zero coefficients).

(8) Find the equation whose roots are the roots of $x^5+7x^4+7x^3-8x^2+x+1=0$ with their sign changed.

Solution:-

The given equation is

$$x^5+7x^4+7x^3-8x^2+x+1=0 \quad \text{—(1)}$$

This equation is complete i.e. there is no missing terms.

So we can find equation whose are all the roots of equation (1) with sign changed by simply changing the sign of every alternate term of one begnning with the second term.

Hence the required equation is

$$x^5 - 7x^4 + 7x^3 + 8x^2 + x - 1 = 0.$$

Q2) change the sign of the roots of the equation $x^7 + 3x^5 + x^3 - x^2 + 7x + 2 = 0$ with their sign changed.

Solution:→

The given equation is

$$x^7 + 3x^5 + x^3 - x^2 + 7x + 2 = 0 \quad \text{——— (1)}$$

This equation (1) is not complete because there are missing terms. We first supply the missing term with zero coefficients.

So complete equation (1) is.

$$x^7 + 0 \cdot x^6 + 3x^5 + 0 \cdot x^4 - x^2 + 7x + 2 = 0$$

∴ required equation is

$$x^7 - 0 \cdot x^6 + 3x^5 - 0 \cdot x^4 + x^2 + 7x - 2 = 0$$

$$\Rightarrow x^7 + 3x^5 + x^2 + 7x - 2 = 0.$$

Ans.

(II) To multiply the roots by a given quantity :→

To transform a given equation whose roots are the given equation multiplied by ~~the~~ a given number $\underline{m}$.

Solution :→ Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the roots of the equation

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \cdots + P_{n-1}x + P_n = 0 \quad \text{—— (1)}$$

so we have identity.

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \cdots + P_{n-1}x + P_n \equiv (x-\alpha_1)(x-\alpha_2)\cdots\cdots(x-\alpha_n) \quad \text{—— (2)}$$

To transform an equation whose roots are $\alpha_1, \alpha_2, \ldots, \alpha_n$ into another whose roots are $m\alpha_1, m\alpha_2, \ldots, m\alpha_n$.

We change x into $\left(\frac{y}{m}\right)$ in the identity (2)

$$\left(\frac{y}{m}\right)^n + P_1\left(\frac{y}{m}\right)^{n-1} + P_2\left(\frac{y}{m}\right)^{n-2} + \cdots + P_{n-1}\left(\frac{y}{m}\right) + P_n$$

$$\equiv \left(\frac{y}{m} - \alpha_1\right)\left(\frac{y}{m} - \alpha_2\right) \cdots \left(\frac{y}{m} - \alpha_n\right)$$

Multiplying throughout by (m^n) we get

$$y^n + mp_1y^{n-1} + m^2p_2y^{n-2} + \cdots + m^{n-1}p_{n-1}y \cancel{+m^np_n}$$

$$+m^np_n \equiv (y-m\alpha_1)(y-m\alpha_2) \cdots (y-m\alpha_n)$$

There is hence. to multiply the roots of a ~~complete~~ compleate equation (If not it may be the compleate by supplying missing terms having ~~zero~~ zero coefficients) by a given quantity 'm' "we have only multiply the succssive coefficient's ~~be~~ beginning with second by $m, m^2, m^3, \cdots m^n$".

(Q1) Change the equation $3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0$ into another equation the coefficients of whose highest term will be unity.

solution: → The given equation is

$$3x^4 - 4x^3 + 4x^2 - 2x + 1 = 0 \quad \text{——— (1)}$$

We have change equation (1) into another equation which co-efficient 3 or x^4 the heighest there will be only so we transform the equation (1) into another equation whose roots multiple of 3 i,e;

We ~~multiplying~~ multiply roots of (1) by 3 which can be done by mearly by multiplying the succssive coefficients ~~begin~~ begnning with second.

$$3,\ 3^2,\ \ldots\ldots$$

$\therefore$ The required transform equation is

$$3x^4 - 4\cdot 3x^2 + 4\cdot 3^2 x^2 - 2\cdot 3^3 x + 3^4 = 0 \quad \text{— (11)}$$

dividing (3) ~~b~~ (2) by 3

$$x^4 - 4x^3 + 12x^2 - 18x + 27 = 0$$

(i,e; coefficient of x^4 is unity)

(3) Remove the fractional coefficients from equation (1)

$$x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - 1 = 0 \quad \text{—— (1)}$$

<u>Solution</u>:→ We have to remove the fractional co-efficients viz, $\frac{1}{2}$ & $\frac{2}{3}$ as L.CM of denominator 2 & 3 of two fraction is 6.

~~So~~ So we multiply the roots of eq^n (1) by (6), mearly multiplying the succssive coefficients begnning with the second by $6, 6^2, 6^3, \ldots\ldots$ so on.

$\therefore$ The required transform equation is

$$x^3 - 6\cdot\frac{1}{2}x^2 + \frac{2}{3}\cdot 36x - 6^3 = 0$$

$$\Rightarrow x^3 - 3x^2 + 24x - 216 = 0$$

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Reciprocal roots and Equation

Let $\alpha_1, \alpha_2, \text{----}, \alpha_n$ be the roots of the equation.

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \text{-----} + P_{n-1}x + P_n = 0 \quad \text{—} (1)$$

so we have identity

$$x^n + P_1 x^{n-1} + P_2 x^{n-2} + \text{------} + P_{n-1}x + P_n \equiv$$

$$(x-\alpha_1)(x-\alpha_2)\text{--------}(x-\alpha_n) \quad \text{—} (2)$$

To transform eq^n (1) into another equation whose roots are $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \text{---}, \frac{1}{\alpha_n}$

(i,e; reciprocal roots of eq^n (1))

We change x by $\frac{1}{y}$ in the identity (2)

$$\frac{1}{y^n}+\frac{P_1}{y^{n-1}}+\frac{P_2}{y^{n-2}}+\cdots\cdots+\frac{P_{n-1}}{y}+P_n$$

$$\equiv\left(\frac{1}{y}-\alpha_1\right)\left(\frac{1}{y}-\alpha_2\right)\cdots\cdots\left(\frac{1}{y}-\alpha_n\right)$$

$$\equiv\frac{1}{y^n}(1-y\alpha_1)(1-y\alpha_2)\cdots\cdots(1-y\alpha_n)$$

$\because$ $\alpha_1, \alpha_2, \ldots, \alpha_n$ are roots of (1), so we have

$$\alpha_1\cdot\alpha_2\cdots\alpha_n=(-1)^n P_n$$

$$\equiv\frac{(-1)^n\{\alpha_1\alpha_2\cdots\alpha_n\}}{y^n}\left(y-\frac{1}{\alpha_1}\right)\left(y-\frac{1}{\alpha_2}\right)\cdots\cdots\left(y-\frac{1}{\alpha_n}\right)$$

$$\equiv\frac{P_n}{y^n}\left(y-\frac{1}{\alpha_1}\right)\left(y-\frac{1}{\alpha_2}\right)\cdots\cdots\left(y-\frac{1}{\alpha_n}\right)$$

Now multiplying that identity by $\frac{y^n}{P^n}$

$$\frac{1}{P_n}+\frac{P_1}{P_n}y+\frac{P_2}{P_n}y^2+\cdots\cdots+\frac{P_{n-2}}{P_n}y^{n-2}+\frac{P_{n-1}}{P_n}y^{n-1}+y^n$$

$$\equiv\left(y-\frac{1}{\alpha_1}\right)\left(y-\frac{1}{\alpha_2}\right)\cdots\cdots\left(y-\frac{1}{\alpha_n}\right)$$

$$\therefore \sum \frac{1}{\alpha_1} = -P_1, \quad \sum \frac{1}{\alpha_1 \alpha_2} = P_2 \cdots\cdots$$

$$\therefore y^n + \frac{P_{n-1}}{P_n} y^{n-2} + \frac{P_{n-2}}{P_n} y^{n-2} + \cdots\cdots + \frac{P_2}{P_n} y^2 + \frac{P_1}{P_n} y$$

$$+ \frac{1}{P_n} \equiv \left(y - \frac{1}{\alpha_1}\right)\left(y - \frac{1}{\alpha_2}\right) \cdots\cdots \left(y - \frac{1}{\alpha_n}\right) \quad -(3)$$

so, $\left(y - \frac{1}{\alpha_1}\right)\left(y - \frac{1}{\alpha_2}\right) \cdots \left(y - \frac{1}{\alpha_n}\right) = 0$ have

roots $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, \cdots, \frac{1}{\alpha_n}$.

Hence if the given equation we replaced x & y and multiply by y^n and resulting polynomial equated to zero.

i.e; equation (3) will have roots the reciprocal of $\alpha_1, \alpha_2, \cdots, \alpha_n$.

~~But if the given equation does~~